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Report

Hydro-mechanical foundation for blood swirling vortex flows formation in the cardio-vascular system and the problem of artificial heart creation

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Abstract

Leonardo da Vinci perhaps was the first who paid attention to the energetic efficiency of existence of vortices emerging near sines of Valsalva and defining normal functioning (opening) of aortal valve. However up to now a fundamental problem of defining of mechanisms of mysterious energetic efficiency of functioning of cardio-vascular system (CVS) of blood feeding of the organism is still remaining significantly not solved and this is, for example, one of the main restriction for the creation of artificial heart and corresponding valve systems. In the present paper, results witnessing possible important role of the very hydro-mechanical mechanism in the realization of the noted energetic efficiency of CVS due to formation in the CVS of spiral structural organization of the arterial blood flow observed by methods of MRT and color Doppler-measuring in the left ventricular of the heart and in aorta (A.Yu.Gorodkov,et.al.).

Keywords Helicity • Vortex flow • Hydrodynamic stability

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Introduction

It is known that spiral structural organization of the fluid flow and also for the blood flow is more stable to turbulent disturbances for significantly greater Reynolds numbers than for pure Hagen-Poiseuille type laminar flow. This stability of swirled flows in CVS may lead to the decreasing risk of thromb formation and to the increase of volumetric transfer though the vessel cross-section in comparison to with more subjected to turbulization flow in which natural spiral structural organization was broken e.g., by surgical interference. In the present paper we show that, observed in many technical and natural systems and phenomena structurally stable organization of the flow of fluid (not only in CVS but in long living vortices of tornado and waterspouts, for example), may follow from variational principles of hydro-mechanics. We also show an important role of the value of spirality or helicity of the flow for the possibility of non-invasive and effective of conducted functional diagnostics of patients. The latter is possible on the base of recording of character periodic variability of pressure pulsations in the flow. On the other side, we give the new theory for linear instability of Hagen-Poiseuille flow (historically first time introduced by Hagen in relation to the very blood flow study) with respect to making spiral resulting flow quasi-periodic disturbances already for not large Reynolds numbers Re>450.

Materials and methods

Extremes of kinetic energy and rate of its dissipation in hydro-mechanics of the swirled flows

In [1] it was obtained the next generalizations of well known hydrodynamic Kelvin's theorem on flow kinetic energy T minimum and Helmholtz's theorem on minimum of the kinetic energy dissipation rate (see below) .

1. First generalization can be obtained by consideration of next condition minimum for functional T

$$T = \rho_0 \int_V d^3x \frac{V^2}{2} \quad (1)$$

under conditions

$$divV = 0 \quad (2)$$

$$S = \frac{\rho_0}{2} \int_V d^3x(\omega V) = inv \quad (3)$$

$$\omega = rotV, \rho_0 = const$$

where S- integral helicity of vortex flow of the fluid with velocity V. In (1)-(3) ρ_0 is the constant dencity of the fluid. The solving of the problem is giving by variation of a functional

$$F = \int_V d^3x[\frac{\rho_0 V^2}{2} + \frac{\mu_1 \rho_0}{2}(\omega V) + \mu_2(x)divV], \quad (4)$$

where μ_1 and μ_2 are Lagrange's multipliers .

From the condition $\delta F = 0$ in (4) we have (5), (6):

$$V = V_0 + \nabla \mu_2 \quad (5)$$

where $\vec{V_0}$ is the flow of Gromeka-Beltrami:

$$rotV_0 = kV_0, divV_0 = 0 \quad (6)$$

In (6) $k = -\frac{1}{\mu_1} = const$

For the particular case (which is important for considerations taking below in paragraph 3) we have from (6) in cylindrical frame of coordinat:

$$V_{or} = 0, V_{oz} = BJ_0(kr), V_{o\varphi} = BJ_1(kr) \quad (7)$$

J_o, J_1 - Bessel functions of zero and first order.

2. For the second variation problem we must consider the variation of the functional

$$F_D = \rho_0 \int_V d^3x [\frac{\nu\omega^2}{2} + \frac{\lambda_1(\omega V)}{2} + \frac{\lambda_2 V^2}{2} + \lambda_3(x) div V] \quad (8)$$

where ν is coefficient of kinematic viscosity. The first variation of (8) is zero for the velocity field (9):

$$\nu rot_1 \omega + \lambda_1 \omega + \lambda_2 V = grad \lambda_3 \quad (9)$$

In particular, for $V = V_0$ from (6) under $k = -\frac{\lambda_1}{2\nu}$ the equation (9) may take place.

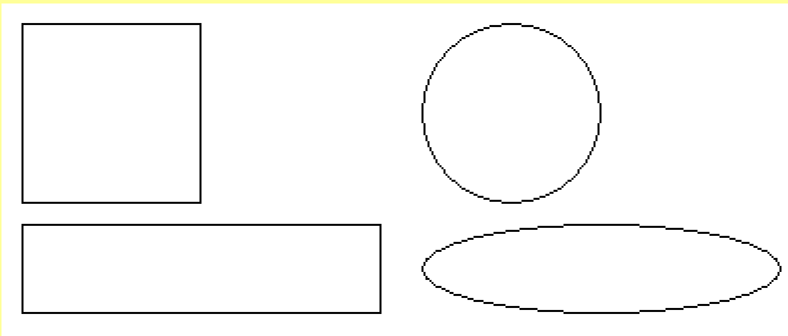

Figure 1. Square and rectangle with equal squares; circle and ellipse with equal squares.

Thus far, among all vortex flows of incompressible fluid having the same kinetic energy (analogous to the square of such figures as rectangles and ellipses as on Fig.1), minimal rate of viscous dissipation of this energy (analogous to the length of perimeter of these geometrical figures) belongs to the homogeneous screw spiral flows of the Gromeka-Beltrami type. It provides observed wide realizability of such type swirled vortex modes in the natural and engineering systems, e.g., in arterial departments of the cardio-vascular system (in the left ventricular of the heart and in aorta.

Maximal volumetric fluid flow rate and "golden" angle of the swirled flow in a pipe

Let's consider (see also [2]) flow of incompressible fluid along the straight not bounded by length pipe with the round cross-section, the same along the entire pipe.

We shall model a near-axis part of the flow in the form of the considered above Gromeka-Beltrami flow (7). In the case when radius of this near-axis core of the flow coincides with the pipe radius R respective volumetric fluid flow rate across round cross-section of a pipe of radius is as follows

$$Q = \frac{2\pi R^2 B J_1(x)}{x}, \tag{10}$$

$$x = kR, Q_0 = \pi R^2 B.$$

In the Fig. 2 below, dependence of the non-dimensional volumetric rate Q/Q_0 on α is presented, i.e., on the angle of inclination of the line tangent to the vortex spiral relative to the pipe axis with

$tg\alpha = \frac{R}{h} = \frac{x}{2}$, and $h = \frac{u_z}{\Omega_z} = \frac{2}{k}$ - is a step of homogeneous-helical vortex spiral.

In the unbounded space $\max Q/Q_0 = 1$ when $\alpha = 0$, since $S = 0$ when $k = 0$

$$S = \frac{(\mathrm{V} rot \mathrm{V})}{2} = \frac{kV^2}{2}. \tag{11}$$

a) Joining of vortex near-axis flow (7) and exact dissipative near-wall solution (wall of the pipe rotates with the angular velocity Ω) when $r = R_1 < R$

$$V_r = 0, V_z = 0, V_\varphi = \frac{\kappa}{r}\left(1 - \frac{r^2}{R^2}\right) + \Omega r. \tag{12}$$

Condition for joining of the velocity field

$$BJ_0(\kappa R_1) = 0$$
$$BJ_1(\kappa R_1) = \frac{\kappa}{R_1}(1 - \frac{R_1^2}{R^2}) + \Omega R_1 \qquad (13)$$

Condition for joining of the pressure field in particular when $\kappa = 0$ is as follows

$$\frac{R_1^2}{R^2} = \frac{1}{2} \qquad (14)$$

i.e. $R_1 \approx 0.707R$ and $x = kR = \gamma_{0,n}, J_0(\gamma_{0,n}) = 0, n = 1,2,\ldots$ $\max Q/Q_0$ when $x = \gamma_{0,1} = 2.404$

It corresponds to the inclination angle of the vortex spiral (approximately equal to the "golden" angle=51.8^0)

$$\alpha = 50.3^0 \qquad (15)$$

Thus far, among all energetically effective (non-dissipative) homogeneous screw spiral vortex modes of the flow of the Gromeka-Beltrami type, only for inclination angles of the spiral close to the "golden" one there may realize maximal volumetric rate-transfer of the fluid across the pipe cross-section. May be, the very hydro-dynamic mechanism is responsible for the formation already at the prenatal stage of ontogenesis of close (with angles about 52 degrees) spiral orientation of the muscle filaments in muscles of the wall of the left ventricular of the heart and in the walls of large arteries.

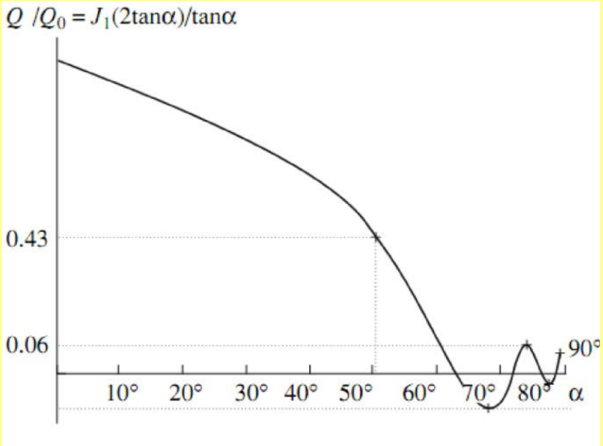

Figure 2. Dependence of the non-dimensional volumetric rate Q/Q_0 on α.

Auto-oscillations emerging when the swirled flow outflows in unbounded space

Let's consider the problem of identification of fluid or blood helicity value by the determine of it's pressure periodic variations. For the purpose of solving this problem we use here the exact solution of the equations of hydrodynamics in a rotating with angular velocity Ω reference system [3]:

$$V_r = -c_0(t)r, V_z = 2c_0(t)z, V_\varphi = \omega(t)r$$
$$p = p_0 + \frac{\rho_0 \omega_1}{2}(r^2 + z^2) \qquad (16)$$

with invariant

$$h = (3c_0^2 + \frac{x^2}{3})x = const \qquad (17)$$

$$x = \omega + \Omega$$

$$y = \frac{x}{(3h)^{1/3}} = \frac{1 - cn(\tau + c_2)}{\sqrt{3} + 1 + (\sqrt{3} - 1)cn(\tau + c_2)} \qquad (18)$$

cn - elliptic Jacobi function, $\tau = 2n^{1/3}t/3^{5/12}; cn(\tau + c_2) \approx \cos(\tau + c_2)$, since $k_0^2 \approx 0.066 << 1$,

$$c_0(t) = \frac{h^{1/3}(1 - y^3)^{1/2}}{3^{2/3}y^{1/2}}, \qquad (19)$$

$$\omega_1(t) = \frac{2h^{2/3}(4y^3 - 1)}{3^{4/3}y}. \tag{20}$$

For consideration of comparison of theory (16)-(20) with experiment [4, 5] (where giving investigation of the pressure pulsation period when the filled swirled flow outflows in unbounded space) lets present (16)-(20) in the next useful form:

$$2\pi f = 0.863(\Omega + \omega(0))[1 + \frac{9}{4}\frac{\gamma^2}{s^2}]^{1/3}$$

$$\Omega = const, \gamma = \frac{r}{\lambda}, s = \frac{(\omega(0) + \Omega)r}{2c_0(0)\lambda}$$

r - radius of the outflow nozzle, λ - length of the confuser, $\frac{\pi}{K(k_0)3^{3/4}} \approx 0.863$, $k_0^2 = 0.066$, f - dominating frequency in the spectrum of pressure pulsations and thus for graphical presentation we have the function:

$f_1 = 0.719[1 + \frac{9\gamma^2}{4s^2}]^{1/3}$ for different γ and s.

In the next figure we have the theoretical dependences for f_1- rigid lines 1, 2, 3, 4 for r/λ $\rightarrow$0.4, 0.7, 1, 1.4.

For experiment [4]: $r/\lambda = 2/7$ $\rightarrow$sign (1), ½ - sign (2), 5/7 – sign (3), 1 – sign (4) and sign (5) in Fig. 3 for experiment [5].

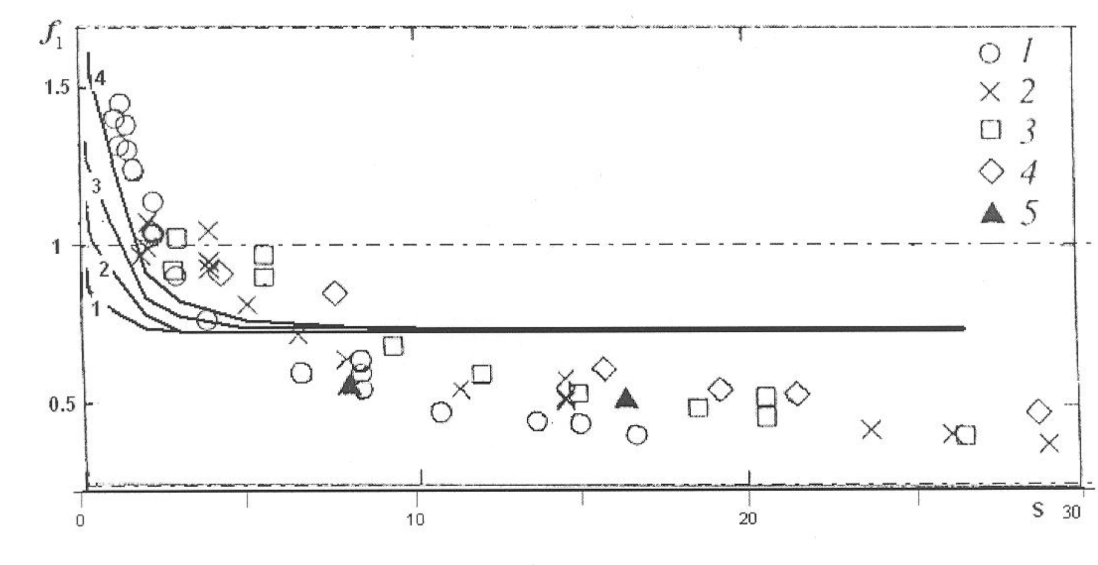

Figure 3. The theoretical dependences for f_1- rigid lines 1, 2, 3, 4 for $r/\lambda \to$0.4, 0.7, 1, 1.4.

Obtained results satisfactory comply with experimental results and may be used for non-invasive diagnostics of the level of helicity (providing stability with respect to the flow turbulization and respective increase of the risk of thromb formation) of the blood flow in the arterial departments of the cardio-vascular system on the base of auscultative measuring of spectrum of frequency of pressure pulsations.

Conclusions

Linear Exponential Instability of the Hagen-Poiseuille Flow with Respect to Synchronous Bi-Periodic Disturbances

The main conclusion of [6, 7] and our paper is that already for Reynolds numbers less than 1000 it is possible to have instability of the laminar Hagen - Poiseuille flow relative to emergence of the spiral resulting flow that is more energetic efficient than other vortex flow with smaller kinetic energy and its rate of dissipation. [6, 7] also give the resolve of fundamental and applied problem of understanding of the mechanism of turbulence development for the Hagen-Poiseuille (HP) [1] flow remains mysterious because of the paradox of the linear stability of this flow to extremely small magnitude disturbances for any large Reynolds number values Re [8-11]. Up to now, to bypass this obvious paradoxical contradiction to experiment, a consensus is made to allow only non-linear mechanism of the HP flow instability to a disturbance with a sufficiently large finite magnitude [12-15]. This assumption is usual (see [10, 11]) based on the very special interpretation of the experiments in which the substantial increase of the threshold Reynolds number Re_{th} value (defining transition from the laminar to turbulent flow state) is achieved by increasing the smoothness of the streamlined pipe surface. In such an interpretation, only correlation between smoothness increasing and resulting decreasing of the initial disturbance magnitude is taken into account. Meanwhile, noted already by O. Reynolds [8] extremely high sensitivity of the value of Re_{th} to the initial disturbance does not exclude possibility of affecting on Re_{th} not only magnitude but frequency characteristics of the disturbances (resulting from non-ideal smoothness of the streamlined surface) as well. Actually, for example, in the experiment [16], it was found that under the fixed magnitude of artificially excited disturbances, HP flow instability develops only in the narrow disturbances frequency range.

Herein, based on the theory [6, 7], we establish, that possibility of the linear absolute instability of the HP flow in the general case also is defined by the value of the additional to Recontrol parameter p, which characterizes frequency properties of the disturbances and affects on the value of $\mathrm{Re}_{th}(p)$ independently from the magnitude of the initial disturbances. Such a parameter p introducing is performed below on the base of pointed by O. Reynolds [8] (and by W. Heisenberg, see [11, 13]) dissipative GP flow instability mechanism related with the action of molecular viscosity ν in the proximity of the solid boundary. According to [8], the mechanism manifests itself as a spontaneous one-stage appearance for $\mathrm{Re} > \mathrm{Re}_{th}$ of the vortexes having character size l_{ν}, «..which does not grow any more contrary to expectations

[1] HP flow is a laminar, stationary flow of the uniform viscous fluid along the static, direct and the length unbounded pipe with the round the same along the pipe axis crosscut.

with the growth of the velocity magnitude [8]». As a result, in addition to the character disturbance scale related to the pipe diameter, $2R$, due to near-boundary vortex-generating viscosity action, there exists a new additional scale l_ν ,which is proportional to the size of some part (which is really activated only for $\nu \neq 0$) of the pipe boundary. It, together with R, can define frequency parameters of the initial disturbances, e.g., the longitudinal along the pipe axis (axis z) spatial periods. The ratio of the periods, $p = \frac{l_\nu}{2R}$, as it is shown below, is a new additional to Re parameter defining the HP flow instability threshold to the conditionally periodic on z extremely small vortex disturbances. Such representation of the disturbances structure meets in observed conditionally periodic Tollmien-Schlichting (TS) waves development of which caused also by near-boundary action of the molecular viscosity. We show, that the assumption of the strict longitudinal (by z) periodicity of the disturbances may result in the paradoxical conclusion on linear stability of the GP flow for any Re with $\mathrm{Re}_{th} \to \infty$. The same time, with the simplest Galerkin's approximation, we get the finite value of the absolute minimum $\mathrm{Re}_{th} \approx 448$ (for $p \approx 1.53..$ -see also complete theory in [6, 7]).

Thus, we have the hydro- mechanical base for understanding the restrictions on the possible modeling of artificial cardio- vascular prototypes. In [17, 18] also considered the important variant of spiral blood flow model with non- zero helicity, which correlated with stability to effects of environmental factors [19].

Statement on ethical issues

Research involving people and/or animals is in full compliance with current national and international ethical standards.

Conflict of interest

None declared.

Author contributions

All authors prepared the manuscript and analyzed the data. S.G.C. drafted the manuscript. All authors read and met the ICMJE criteria for authorship. All authors read and approved the final manuscript.

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